# What Time Is It?

## Telling time with a homemade sundial

### A Note to Parents

This project is written, during the new coronavirus pandemic, for primary school children to do at home with minimal assistance from their parents. As parents you will need to help your children get started and offer support, perhaps some reading, interpretation and personal interest as appropriate. All children are different. But you may find some of the things interesting yourselves.

The sundial is made with a straight stick or dowel and small stones because these are things with which young children are familiar and can be found without going to a store. Also, they are waterproof, so it won't matter if it rains. This assumes that there are sticks and small rocks and a spot of earth in the neighborhood. This makes it easy to do in rural areas and at homes with yards.

If you are doing this project in an urban neighborhood or apartment house with no open ground, sticks or stones, you may make the sundial on pavement by sticking a pencil, or chop stick, skewer, straw, or wire clothes hanger etc. at about a 45 degree angle in a ball of clay or taping it at about a 45 degree angle to a brick, block of wood, coffee cup, etc.; you get the idea. The angle makes a longer shadow for something as short as a pencil, and is more important the closer you are to the equator. ***For details on this urban variation, and enhancements for older students, please see the Appendix beginning on page 18 of this document.***

This project can be of interest to older students. In this case, even when pounding a stick or dowel into the ground, they should try to angle it toward the north in the northern hemisphere, or toward the south in the southern hemisphere (a vertical stick is easier for young children.). They can use a compass, cell phone, or the North Star or Southern Cross at night, or just guess as close as possible with advice from an adult. Not all science has to be perfect; just good enough for the purpose at hand.

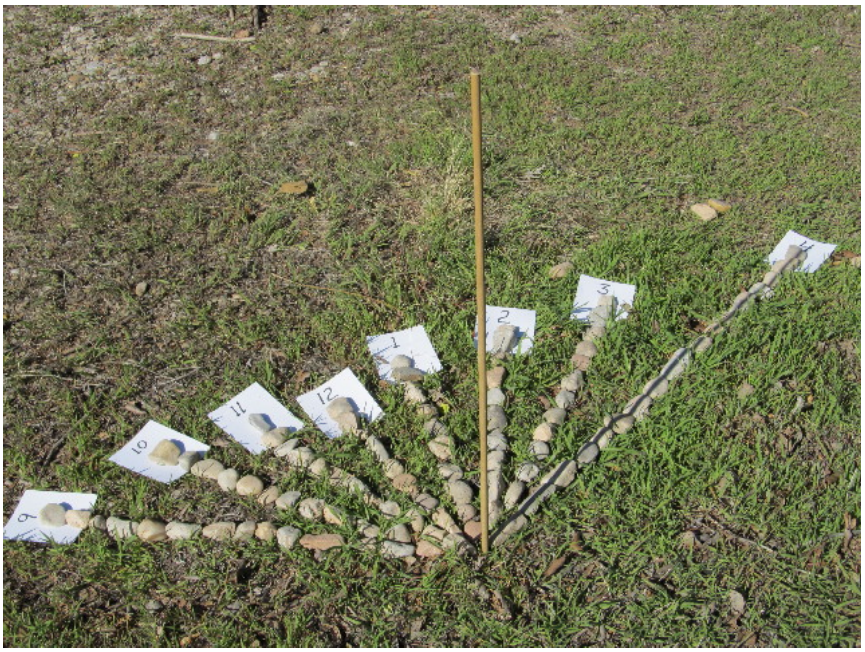
The following guide and commentary make some digressions into history, math and language when they are relevant or related to the main activity. This is a normal, human way of inquiring, and not a way that most school curricula are designed. But doing this supports the common and social way that children inquire about the world, and it demonstrates how so many topics and things in the world share threads of relationship and interconnection.

The following project can be a happy and interesting experience for the whole family.

# What Time Is It?

## Telling time with a homemade sundial

Making a backyard sundial is a great math and science experiment, because it relates the mathematics of time with the science of the solar system. It is easy to make, and it leads to surprising observations and insights. And the passage of daytime with the passage of the sun is a source of human emotion, myth and poetry.



Can you see the shadow of the stick going over the line of rocks on the far right leading to the number 4?

Materials:
a ruler or tape measure

a dowel or straight stick about 1/4 to 3/8 inch thick, and at least 24 inches long

a bunch of small stones about 1/2 to 1 inch wide

ten 3 by 5 inch pieces of paper, or 3 by 5 inch index cards

a waterproof marking pen

printed Sundial Data Table, Sundial Graph Paper, clock faces, found on pages 15-17

**Step 1.** Spend one day finding a sunny spot in your yard where the sun shines from about 9 o'clock in the morning until about 4 or 5 o'clock in the afternoon.

**Step 2.** The next day, if it isn't raining, go to the middle of your sunny spot and pound your dowel or straight stick about 6 inches into the ground with a big rock or a hammer. Use your ruler or tape measure to make sure about 18 inches of your stick is above the ground and that it is straight up and down.  Can you see the shadow that the stick is making on the ground?

The thing that makes the shadow on a sundial is called a gnomon (pronounced no-mon).  Your gnomon is your dowel or stick.  The oldest gnomon ever found was 4,300 years old.  It was found in China.  Our word for it, gnomon, is an ancient Greek word meaning 'one that knows' or examiner or interpreter.  For us it is a 'thing that lets us know' what time it is.

Measure the height of your gnomon and write it on your Sundial Data Table.

Next, collect about 100 small stones each about 1/2 to 1 inch wide and pile them up about 36 inches away from your stick.

**Step 3.** The next day at 9 o'clock in the morning, if the sun is shining, go out to your sundial and make a line of stones all along the shadow that your sundial is making on the ground.  Stop making the line of stones when you reach the end of the shadow.

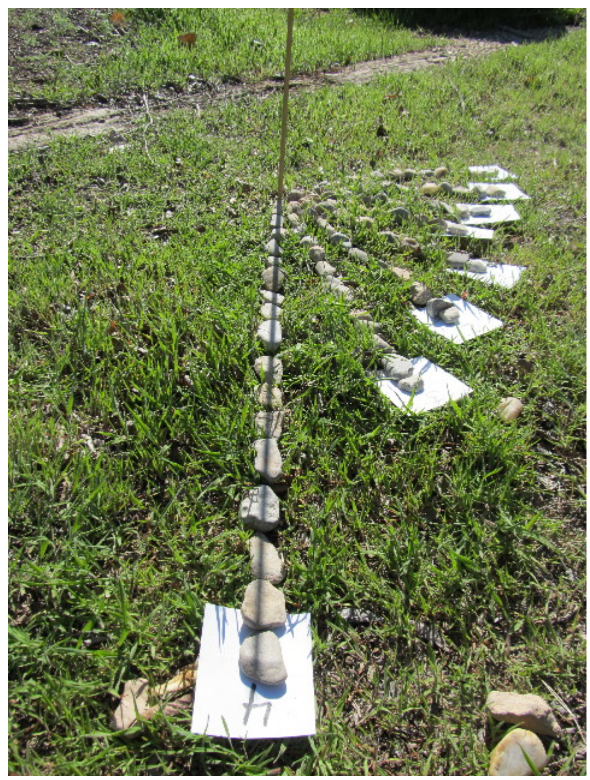
Do you see the dark line
of the shadow
that the gnomon is making
on top of the stones
of my sundial?

Use your ruler or tape measure to measure the length of the shadow your gnomon has made.

It is the same as the length of your line of stones.

Write the length of the shadow next to the 9 o'clock space on your Sundial Data Table.

Write the time, 9, on one end of a 3 by 5 inch piece of paper, and write the length of the shadow under the time, and put the paper at the end of your line of stones.



**Step 4.** One hour later at 10 o'clock in the morning, if the sun is shining, go out to your sundial and make a line of stones all along the shadow that your sundial is making on the ground.  Stop making the line of stones when you reach the end of the shadow.

Use your ruler or tape measure to measure the length of the shadow your gnomon has made.  It is the same as the length of your line of stones.

Write the length of the shadow next to the 10 o'clock space on your Sundial Data Table.

Write the time, 10, on one end of a 3 by 5 inch piece of paper, and write the length of the shadow under the time, and put the paper at the end of your line of stones.

**Step 5.**  One hour later at 11 o'clock in the morning, if the sun is shining, go out to your sundial and make a line of stones all along the shadow that your sundial is making on the ground.  Stop making the line of stones when you reach the end of the shadow.

Use your ruler or tape measure to measure the length of the shadow your gnomon has made.  It is the same as the length of your line of stones.

Write the length of the shadow next to the 11 o'clock space on your Sundial Data Table.

Write the time, 11, on one end of a 3 by 5 inch piece of paper, and write the length of the shadow under the time, and put the paper at the end of your line of stones.

**Steps 6, 7, 8, 9, and 10.**  Every hour during the rest of the day, at 12 o'clock, 1 o'clock, 2 o'clock, 3 o'clock and 4 o'clock make a line of stones all along the shadow that your sundial is making on the ground.  Stop making the line of stones when you reach the end of the shadow.

Use your ruler or tape measure to measure the length of the shadow your gnomon has made.  It is the same as the length of your line of stones.

Write the length of the shadow next to the time space on your Sundial Data Table.

For example, this is what my sundial data table looks like.

## Sundial Data Table

Date: March 28, 2020

Where is your sundial? next to the driveway on the east side of my house

Height of your gnomon: 18 inches

| Time of day | Length of shadow cast by the gnomon |
|---|---|
| 8 o'clock AM | I did not go out at 8 o'clock |
| 9 o'clock AM | 16 inches |
| 10 o'clock AM | 13 and 1/4 inches |
| 11 o'clock AM | 11 inches |
| 12 o'clock NOON | 9 and 1/2 inches |
| 1 o'clock PM | 11 inches |
| 2 o'clock PM | 13 inches |
| 3 o'clock PM | 16 inches |
| 4 o'clock PM | 26 inches |
| 5 o'clock PM | I did not go out at 5 o'clock |

Write the time on one end of a 3 by 5 inch piece of paper, and write the length of the shadow under the time, and put the paper at the end of your line of stones.

**Experiment.** If you want to try an experiment, make a shorter gnomon that is only about 9 inches long above the ground. Then during a sunny day measure the length of the shadows every hour and write the lengths on a new Sundial Data Table. Compare these lengths with the lengths from the first data table. What do you notice?

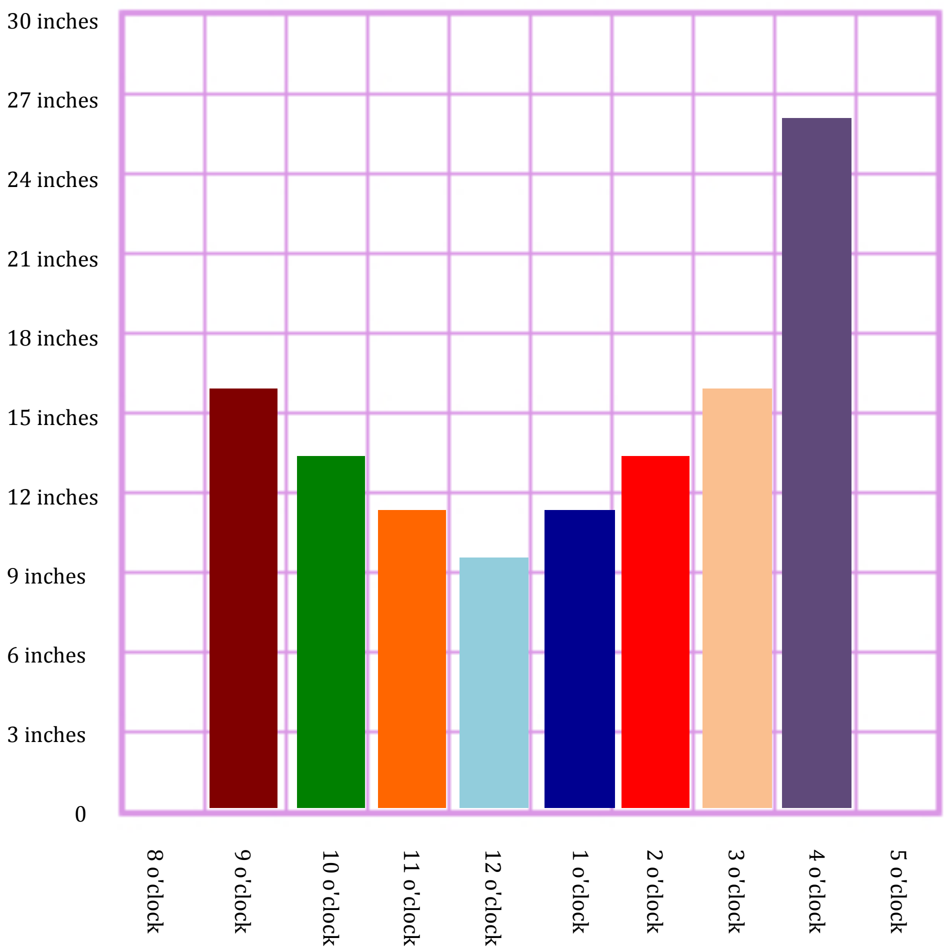
**Step 11.** Using your sundial graph paper, make bar graphs of the length of each hour's shadow. Get the lengths for each hour from your Sundial Data Table. If the length of your shadow is different than one of the numbers going up the left side of your graph paper, estimate the number that is closest to your measurement. Each square going up the graph represents 3 inches.

This is an example of the bar graph of my sundial shadows.

Sundial Graph Paper

Title: Paul's sundial shadow length bar graph

Height of gnomon: 18 inches




Why do you think that the 4 o'clock shadow is so much longer than the other shadows?
Why do you think that the 9 o'clock shadow and the 3 o'clock shadow are the same length?
Why do you think the 11 o'clock shadow is the same length as the 1 o'clock shadow?

**Step 12.** The next day, if the sun is shining, go outside to your sundial and see where the shadow of your gnomon is. Just by looking at the shadow of your sundial, can you tell what time is it? Then do something else, and after a while go back to your sundial. What time is it now?

**Step 13.** The next day, print out the clock face and cut it out. Then at about 9 or 10 o'clock in the morning take it out to your sundial and line it up so that the numbers on the edge of the clock face each are sitting on top of the matching line of stones. So the 9 of the clock face is on the 9 o'clock line of stones and at the same time the 2 of the clock face is on the 2 o'clock line of stones. [You can see that I had to fold my clock face so it fit next to the gnomon and the numbers still matched the lines of stones.]

What time does the shadow of your gnomon say it is?

Here is what my clock face looks like lined up with my lines of stones. I folded the clock face to fit next to the gnomon, and put a rock on it so the wind would not blow it away. The shadow of the gnomon shows that it is a little past 3 o'clock, or about 3:18.


**Step 14.** A few times during the day, go out to your sundial and write down what time the shadow of your gnomon on your clock face says it is. You can round to the nearest hour, so, for example, if the shadow is between 3 and 4, but it is closer to the 3 o'clock than to the 4 o'clock, you can write that it is about 3 o'clock.

If you want to be exact, each of the marks between the hour numbers represents 12 minutes. So my clock face shows half way between the 12 minute mark and the 24 minute mark after 3 o'clock, which is 18 minutes past 3 o'clock, or 3:18 o'clock.

**Step 15.** Why do we have a clock with 12 hours instead of something easier like 10, and with 12 minute marks? And with 60 minutes in an hour instead of 100?

Well, it is a very old habit, thousands of years old, and old habits are hard to break. The oldest 12-hour clocks were found in ancient Egypt and ancient Babylonia in Mesopotamia. An Egyptian **sundial** clock for daytime and an Egyptian water clock for nighttime were found in the tomb of Pharaoh Amenhotep I. The sundial and water clock are about 3,500 years old, and they divided the times of use into 12 hours each.

Using the number 12 for time probably started before the ancient Egyptians, when even more ancient astronomers discovered that there were usually 12 full moons in a solar year. (About every three years there are 13 full moons.) A solar year is the number of days, about 365 days, it takes for the sun to rise the next time in the same place on the horizon on the shortest day of the year or the longest day of the year.

These days are called the winter solstice and the summer solstice. The word solstice comes from Latin words meaning 'sun stoppage.' These two days were important for ancient people because the sunrise seemed to stop moving north or south on the horizon at that time, and started moving back the other direction. And days started getting longer or shorter depending if you were north or south of the equator.

The ancient people thought the sun moved across the sky during the day, which is why the gnomon shadow on their sundials (and on your sundials) changed lengths during the day. Thousands of years later, astronomers figured out that the sun does not move in relation to the Earth, but instead the Earth turns in relation to the sun. As the Earth turns it looks like the sun is moving; it is an illusion. It looks that way because the Earth is turning while the sun is standing still in relation to the Earth.

**So your sundial works because the Earth turns!**

**Step 16.** You can make an example of this with a flashlight and a pencil. This works best at night. Go to a dark room in your house. Turn the flashlight on and put it on a chair or a table so it is facing you. Then stand in front of the flashlight.

Pretend that the flashlight is the sun. It is lighting up your shirt on the front of your body, but it is not lighting up your shirt on your back. It is daytime on the front of your shirt and nighttime on the back of your shirt. Look around at your shirt under your arm. There is hardly any light from the flashlight shining there.

Now hold the pencil so that it is sticking straight out from your belly toward the flashlight .... I mean the sun.

Pretend that your pencil is a gnomon. And pretend that you are the Earth. While looking at your shirt, start slowly turning around. Is the shirt on one arm getting more and more light? Is the shirt on your other arm getting more and more dark? Is the pencil gnomon making a shadow? As you slowly turn, what is happening to the gnomon shadow?

When you have turned half way around it is night on the front of your shirt and daytime on the back of your shirt. As you keep turning to face the flashlight ... I mean sun ... again, the front of your shirt is daytime again. And what happened to the shadow of your pencil gnomon?

If you keep turning around over and over your front ... I mean Earth ... keeps changing from day to night to day to night, just as a place on the Earth keeps changing from day to night to day to night, over and over because the Earth never stops turning.

**Step 17.** OK, back to why we have 12-hour sundials and 12-hour clocks with 60 minutes in an hour and 60 seconds in a minute.

"The origin of our time system of 24 hours in a day with each hour subdivided into 60 minutes and then 60 seconds is complex and interesting," says Dr. Nick Lomb, from the Sydney, Australia Observatory.

**Ancient Egyptians: 12-hour clock.** "Our 24-hour day comes from the ancient Egyptians who divided daytime into 10 hours they measured with devices such as shadow clocks [**sundials**], and added a twilight hour at the beginning and another one at the end of the daytime [10 + 1 + 1 = 12 hours]," says Lomb. "Nighttime was divided in 12 hours, based on the observations of stars."

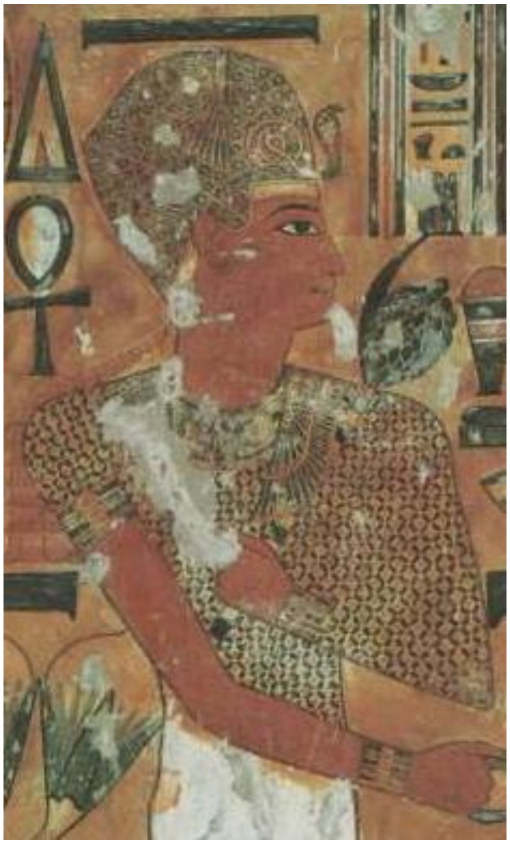
The ancient Egyptians measured nighttime with 12-hour water clocks. Remember that the number 12 was special because there usually are 12 full moons in a year. The word 'month' comes from the word 'moon': moonth. There are 12 months in a year.

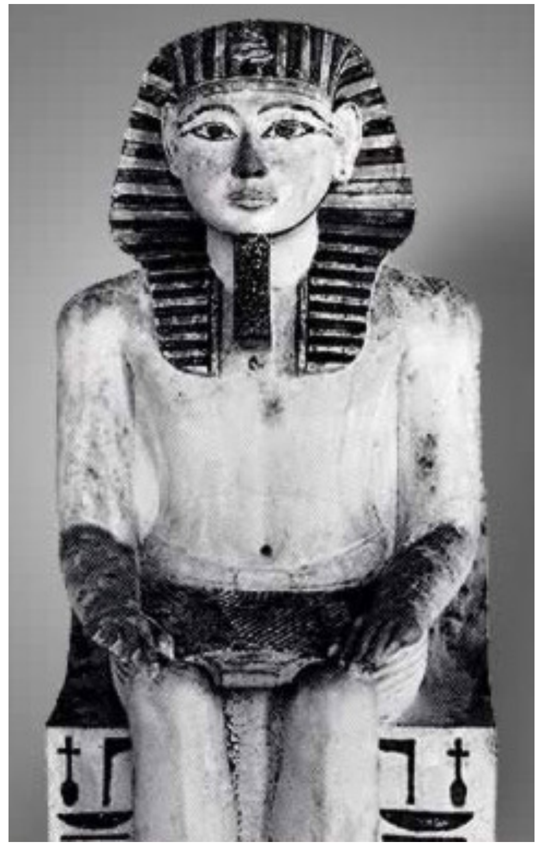
**Ancient Babylonians: hours, 60 minutes, 60 seconds.** "The subdivision of hours and minutes into 60 comes from the ancient Babylonians who had a predilection for using numbers to the base 60. We have retained from the Babylonians not only hours and minutes divided into 60, but also their division of a circle into 360 parts or degrees," says Lomb.

Lomb says that probably the Babylonians were interested in the number 360 because that was their estimate for the number of days in a year.

**Ancient Chinese.** The ancient Chinese also used a time system where they divided the day into 24 hours, originally with the middle of the first 12 hours being at midnight and the middle of the second 12 hours being at noon. So all over the world, because of studying the sun and the moon for thousands of years, people have been interested in the numbers 12, 60 and 360. 5 x 12 = 60 and 6 x 60 = 360.

This is the Egyptian Pharaoh Amenhotep I, who lived 3,500 years ago.
A sundial and a water clock were found in his tomb.





This is an ancient Egyptian sundial.
Do you see the hole for the gnomon?
Can you see how it is like your sundial?

This is an ancient Egyptian water clock.
It was filled with water, with a small hole at the bottom where water drops dripped out.


This is the Babylonian King Hammurabi, who lived 3,700 years ago.

Babylonian astronomers estimated there were 360 days in a year. They divided an hour into 60 minutes. They wrote in cuneiform (pronounced kyou-nee-ih-form), which is the writing Hammurabi is looking at in the carving at left.

Writing was invented about 5,000 years ago, in Mesopotamia south of Babylon in Sumer, in the southern part of present-day Iraq.

Independently, it also was invented at about the same time in Egypt and then in China and Mexico. These four places in the world each had different ways of writing. That is because people in these places invented their own way to write. However, it's interesting that these look kind of similar even though the people didn't know about each other's writing.

Babylonia cuneiform

Egypt hieroglyphs

China xiaozhuan

Mexico Olmec

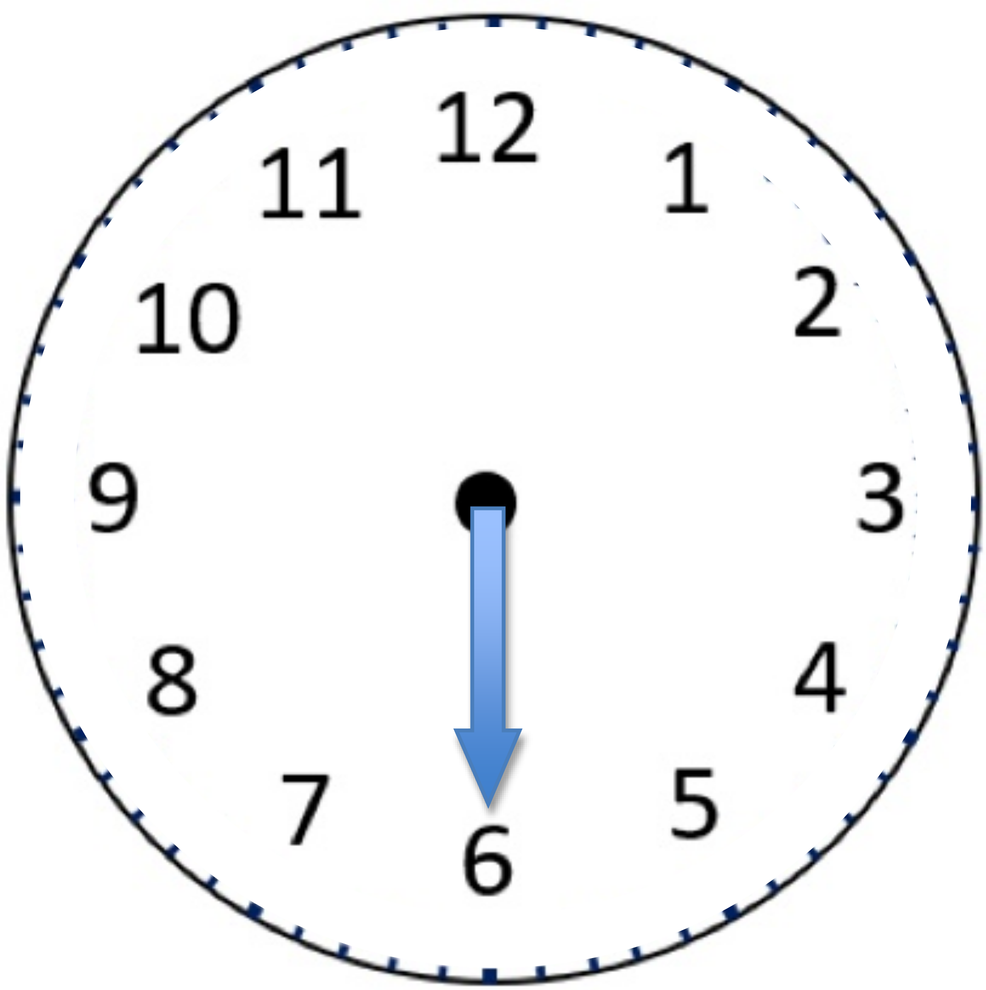
If you go to this museum website, you can have your name translated into cuneiform: https://www.penn.museum/cgi/cuneiform.php

**Step 18.** Why does our sundial clock have 5 marks between the hours that are each 12 minutes?

Well, it is because a sundial has only one shadow made by only one hand (the gnomon). So between the hours it tells parts of an hour that are 1/5 of an hour. 1/5 of 60 minutes in an hour is the same as 60 ÷ 5 = 12 minutes. But why are there 5 parts to an hour? (Count them; there are five spaces between each number from one hour to the next.)

It is because the clocks that we use at home or at school have two clock faces on top of each other. One is a 12-hour clock face with a short hand for telling hours, and the other is a 1-hour clock face with a long hand for telling minutes.

Here is what the 12-hour clock face looks like for telling hours with the short hand. It shows 6 o'clock, which is six hours after the hour that ends at 12. So this clock counts 12 hours if it goes all the way around the circle, which is half a day that starts at midnight and ends at noon, when the clock starts over from noon to midnight.



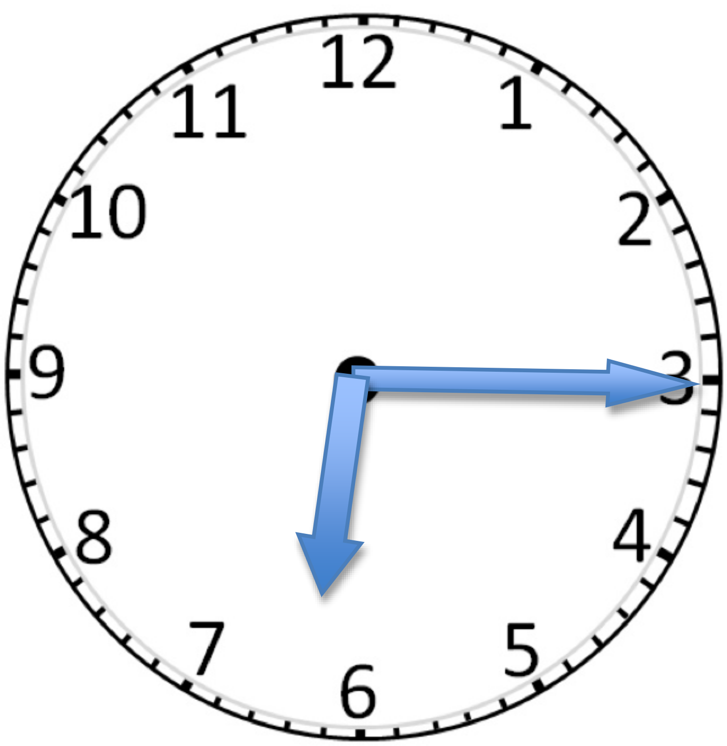
Here is what the 1-hour clock face looks like for telling minutes with the long hand. It shows 15 minutes after the hour that starts at 00. The 00 can also be a 60, so this clock face counts 60 minutes in an hour if it goes all the way around the circle.


The first mechanical clocks only had an hour hand with a 12-hour clock face. But after the invention of accurate pendulum clocks, around the year 1690, clockmakers were regularly adding a minute hand to clocks. So they added a 1-hour clock face on top of the 12-hour clock face. The new face might have looked like the following one.

A clock with both a 12-hour face showing hours and a 1-hour face showing minutes. This clock is showing 15 minutes after 6 o'clock. Another way to say this is 6:15.


But for most people this had too many numbers, so the new clocks looked like this.

**It's 6:15!**



# Sundial Data Table

Date: ________________________

Where is your sundial?________________________________________________________________

________________________________________________________________________________________

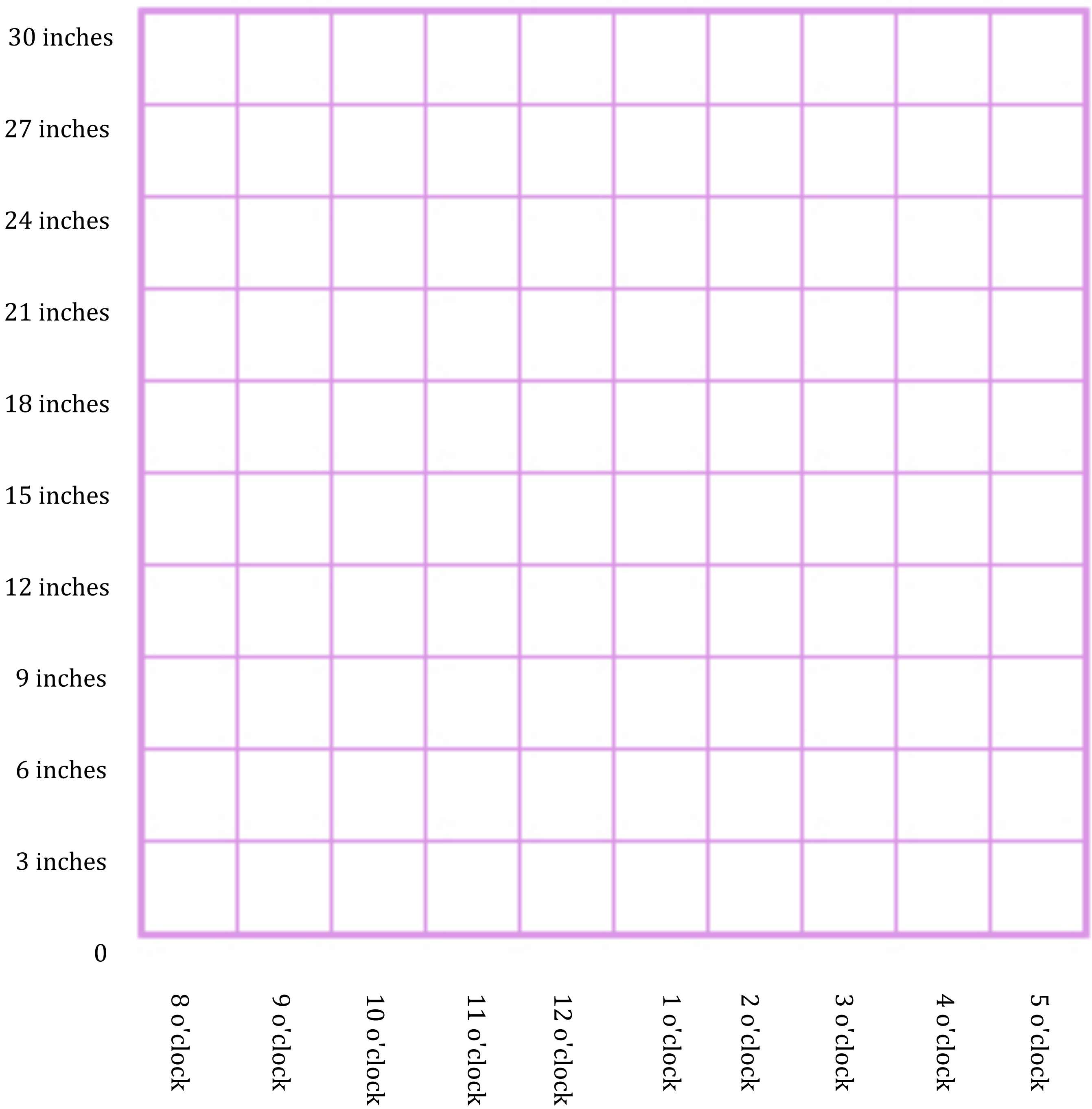
________________________________________________________________________________________

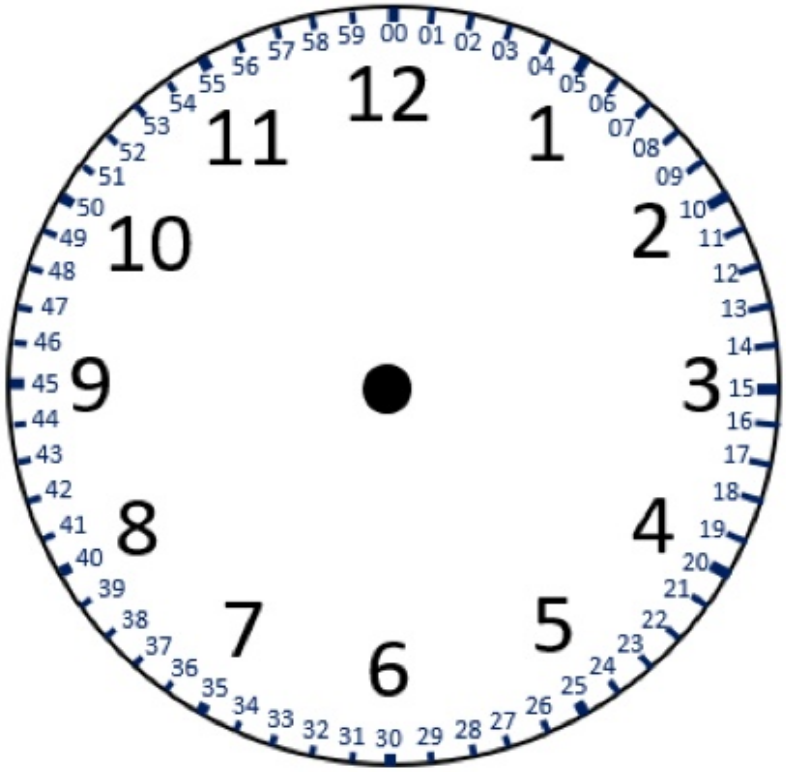
Height of your gnomon: ___________________________________________

| Time of day | Length of shadow cast by the gnomon |
| --- | --- |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

# Sundial Graph Paper

Title:______________________________________________

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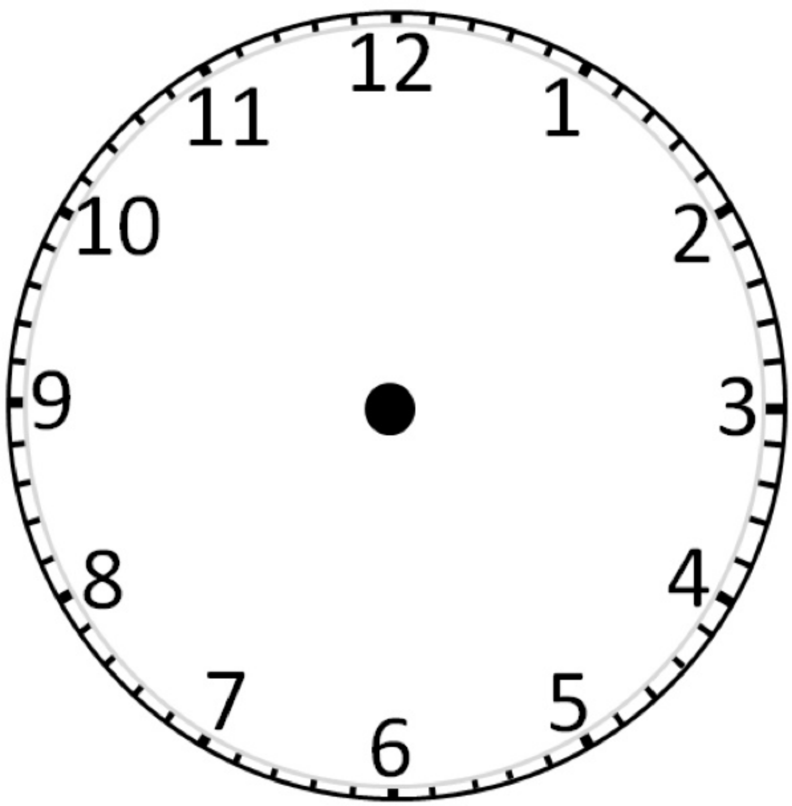
00 01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59


12
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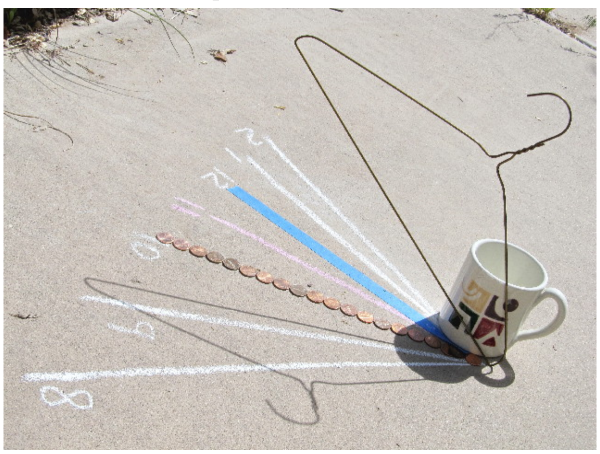
**APPENDIX:**
***What Time Is It?* and *Where Am I?***
**for urban and older students**

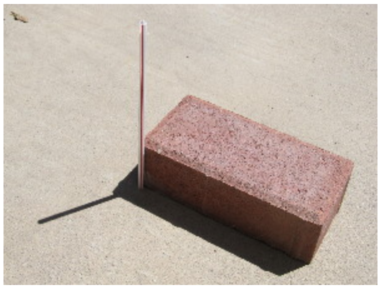
If you are doing this project in an urban neighborhood or apartment house with no open ground, sticks or stones, you can make the sundial on pavement by sticking a pencil, or chopstick, skewer, straw, or wire clothes hanger, etc., as a gnomon at about a 45-degree angle in a ball of clay or taping it at about a 45-degree angle to a brick, block of wood, coffee cup; you get the idea. The angle makes a longer shadow for something as short as a pencil, and is more important the closer you are to the equator.

Aim the pencil, chopstick, clothes hanger, etc., toward the north if you are in the northern hemisphere, or toward the south if you are in the southern hemisphere. An approximation of north or south is fine for this project. Instead of stones, you can trace the shadows on the pavement with a piece of chalk, or lay down tape under the shadows, or if you find a protected space open to the sky you can use beans or coins.

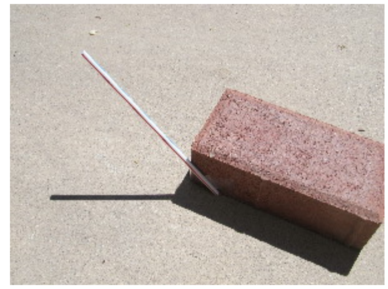


With older students, even when pounding a stick or dowel into the ground, they should try to angle it toward the north (A vertical stick is easier for young children.). Anywhere between a 30-degree and 45-degree angle from vertical is adequate. They can use a compass, cell phone, or find the North Star at night, or just guess as close as possible with advice from an adult. Not all science has to be perfect; just good enough for the purpose at hand.

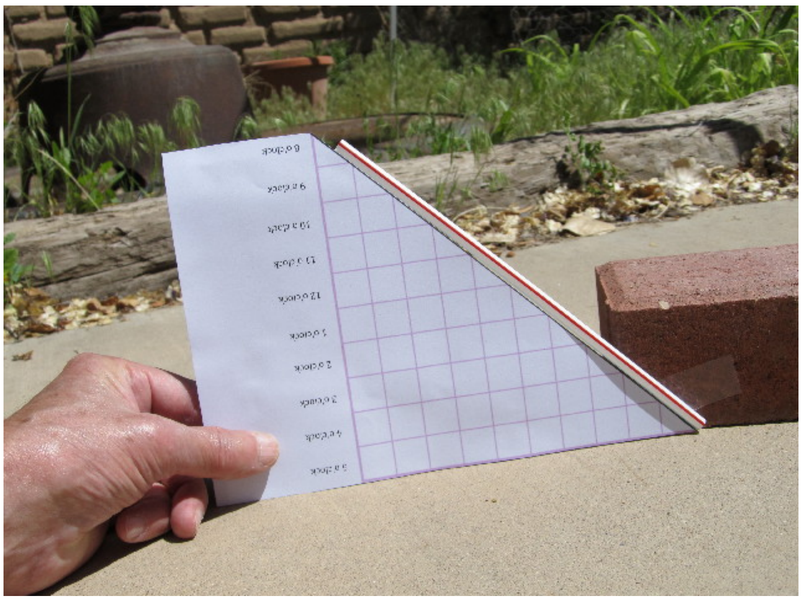
You can see that the chopstick stuck vertically into the clay on the left makes a shorter shadow than the chopstick stuck at a 45-degree angle into the clay on the right.



You can see the same difference in the shadows with a drinking straw taped vertically to the brick on the left and a straw taped at a 45-degree angle to the brick on the right.

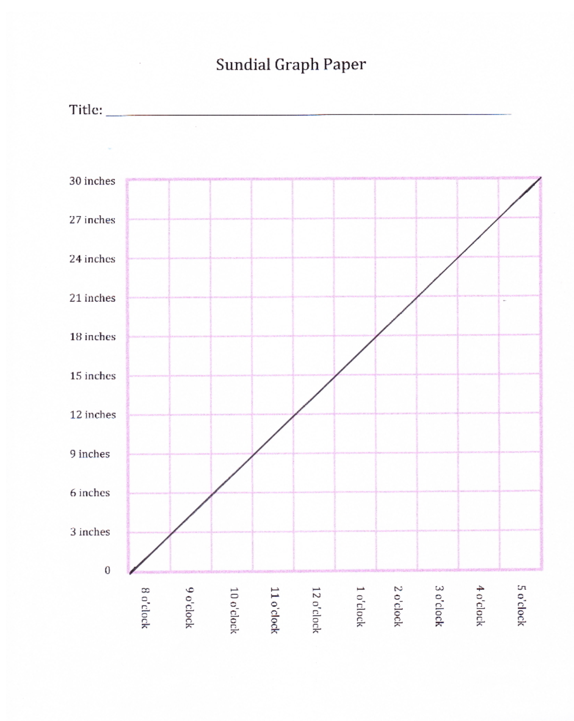


You can measure a 45-degree angle by cutting a blank sundial graph paper in half on the diagonal.



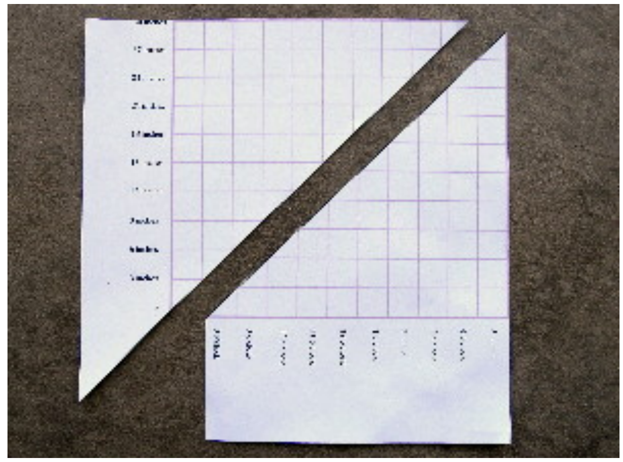
Here's how.

Print a copy of the Sundial Graph Paper found back on page 16. Using a ruler or straight edge, draw a line from the lower left corner of the graph paper (the zero point) to the upper right corner of the graph paper as shown here. --------->




Mathematicians and carpenters and way-finders call this line a diagonal.

Then cut along the top red line and cut along the right side red line, and then cut along the diagonal, and you get two pieces of graph paper that look like this. ------------>



Now you can use one of the triangles to align you gnomon (pencil, straw, clothes hanger, etc.) to a 45-degree angle, as you can see in the photo on the previous page.

**[Now go back to page 2 Step 1 and follow the Steps to mark, measure and graph the length of your shadows, and return here after finishing page 14.]**

Why is this a 45-degree angle, you may ask? Why does this angle follow a line between opposite corners of a square? And why is it called a diagonal? Diagonal is from a Greek word *diagonios* meaning from angle to angle (corner to corner).

Do you remember that the ancient Babylonians divided a circle into 360 parts we now call degrees? And they probably got that idea from ancient astronomers that found there were about 360 days in a year. They also noticed that the sun and moon seem to follow a half circle across the sky.

But another reason was that Babylonian mathematicians really liked the number 360, because it's special. 360 can be evenly divided by 24 different numbers! That's more than any other number less than 360. These numbers are 1, 2, 3, 4, 5, 6, (not 7), 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, and 360.

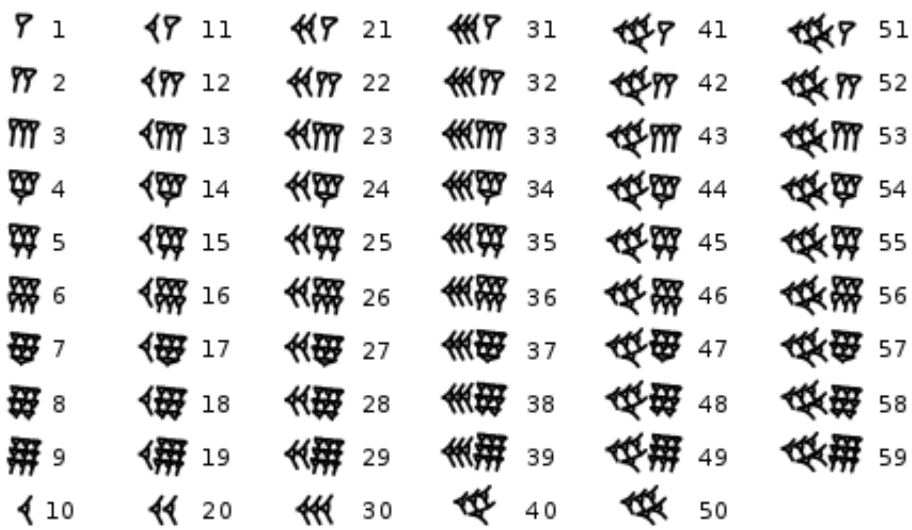
If you multiply the first and last number of this list you get 1 x 360 = 360. And if you step in one number from each end and multiply those numbers you get 2 x 180 = 360. And if you keep stepping in and multiplying you always get 360. 3 x 120 = 360; 4 x 90 = 360; 5 x 72 = 360; etc.

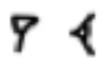
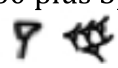
This is true of any list of divisors. For example, 100 can be divided by 9 different numbers. These numbers are 1, 2, 4, 5, 10, 20, 25, 50, and 100. 1 x 100 = 100, 2 x 50 = 100, and so on.

In addition, you will notice that 360 can be divided by both 2 and 3. 100 can't be divided by 3 (without making a fraction). All the other numbers less than 360 that can be divided by both 2 and 3 are divisors of 360, like 12. Can you figure out what the others are? What's neat about this is that 360 can be divided into 2 parts, or 3 parts, or 4 parts, or 6 parts, or 12 parts, etc. It also can be divided into 5 parts.

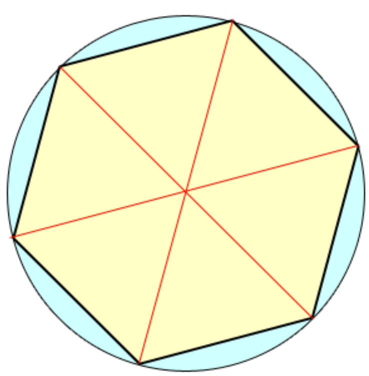
On top of that, the Babylonians used a base 60 number system. We use a base 10 number system, which means that when we write numbers we use only ten numerals or symbols: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. Every time we go past 9 to 10, we add a zero and start over from 1.  So, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20.  15 means one 10 plus 5. 20 means two 10s.  30 means three 10s.  So in a base 10 system we count by tens.

The Babylonians counted by 60s.  So if they used our numerals or symbols for numbers, they would write 10 to mean one 60.  15 would mean one 60 plus 5.  20 would mean two 60s.  30 would mean three 60s.  And so on.  But the Babylonians used their own, cuneiform numerals or symbols that looked like this:
(Photo: Josell7/Wikimedia Commons)




So 65 might look like this:  (one 60 plus 5).  And 70 might look like this:  (one 60 plus 10).  100 might be  (one 60 plus 40).

OK, now back to geometry.



There is more fun math involved with the number 360. If you draw a regular hexagon inside a circle that touches all of the hexagon's corners (*hex* is from an ancient Greek word meaning 6, so a hexagon is a 6-sided figure), and then draw lines between the opposite corners of the hexagon through the center of the circle, you get 6 triangles.

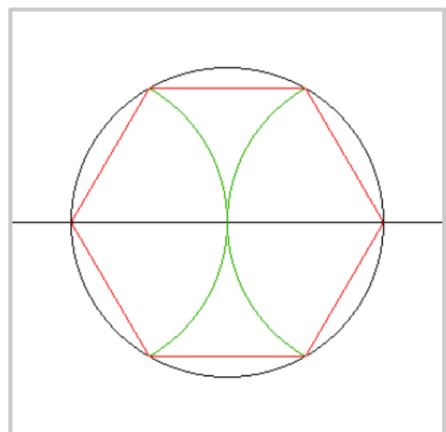
It turns out that each side of these triangles is exactly equal in length to every other side of these triangles.  So the six black lines around the hexagon are each equal in length to the six red lines from the center of the circle to the corners of the hexagon.

Imagine how happy the Babylonian mathematicians were when they saw exactly 6 identical triangles inside a circle; especially because they counted by 60s!  They were interested in the geometry of circles and in measuring the distance that the moon and sun traveled across the sky in what looked like a semicircle (*semi* is a Latin word meaning half.). A semicircle is half of a circle that has been divided by its diameter [from a Greek word *diametros*, *dia*- meaning through and *metron* meaning measure].

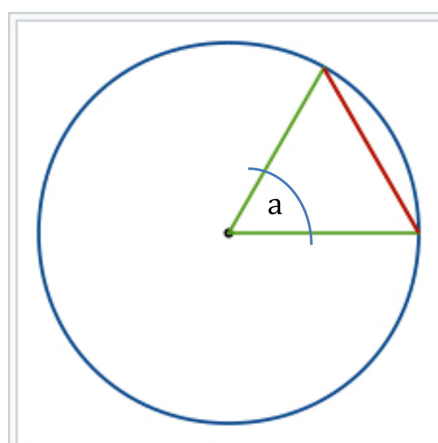
A diameter of a circle is a straight line segment through the center of the circle and ending at each side of the circle. The relationship between the distance around a circle called a circumference and the length of its diameter will become an important mathematical constant, a number that doesn't change no matter the size of the circle.

So it might have been that when the Babylonians saw the hexagon with its 6 triangles inscribed in a circle with its diagonal lines exactly the same as diameters of the circle, and that they counted by 60s and knew that a year had approximately 360 days (which was 6 x 60), they decided to divide the circumference of a circle into 360 steps that we now call degrees (*degree* is from a Latin word meaning step). So then each of the triangles measured 60 degrees of the circumference. How beautifully this all fit together, with the 6 and 60 and 360, and the half of 360 = 180 degrees in a semicircle!

We still use these measurements today, almost 4,000 years later.



The drawing to the right is from an animation of how the ancient mathematicians might have drawn a regular hexagon inside a circle. To see this animation, go to the following link. You may need to copy and paste it in your browser. (*link* to this animation by Aldoaldoz - *https://commons.wikimedia.org/wiki/File:Regular_Hexagon_Inscribed_in_a_Circle.gif#/media/File:Regular_Hexagon_Inscribed_in_a_Circle.gif*)




This drawing shows one of the six triangles. You can see that the red line is the same length as each of the green lines. The three sides of this triangle are equal to each other, which is why mathematicians call this kind of triangle an equilateral triangle (*lateral* is from a Latin word meaning side). The green sides of this triangle mark off 60 degrees of the circle, so the size of the angle marked 'a' is also 60 degrees. The Babylonians made 360 much smaller angles from the center of the circle so that each one marks off 1 degree along the circumference. These could be used to measure the sun's location each day.

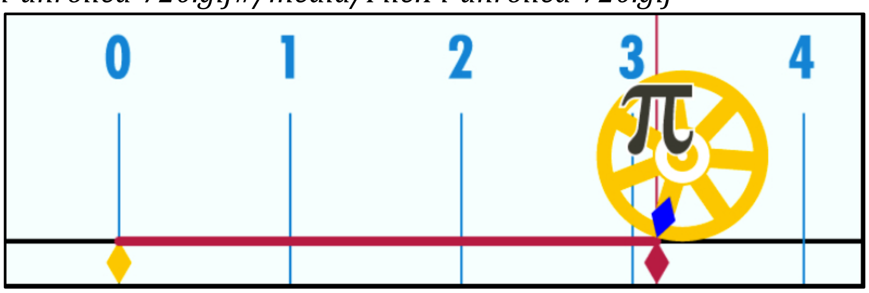
The Babylonians probably were surprised that every time they divided the circumference of any-sized circle, by its diameter, they always got almost exactly the same number. This made them think that it really was exactly the same number, and they got slightly different numbers because their measurements weren't exact. So they spent a lot of time trying to figure out what that number was.

We call that number Pi [or $\pi$, a Greek letter pronounced pie]. It is the circumference of a circle divided by the circle's diameter. Another way to say this is that Pi is the ratio of the circumference of a circle to its diameter. How many diameters of a circle equal the circumference of that same circle? It's 3 and a little more, or 3.14159....

Ancient mathematicians did not have positional decimal fractions, which we think were used the first time in Syria and Iraq by Arab mathematician Abu'l-Hasan al-Uqlidisi about 1,000 years ago. So they had to use fractions with whole numbers.

Below is an animation of one way to imagine a value for Pi ($\pi$). Measure a unit equal to the diameter of a circle [The distance between each blue number below is one diameter.], and then starting with the diameter on zero, roll the circle (the wheel) one complete rotation (one complete turn). See where the blue arrow on the circle in the animation touches the bottom line after it goes around? No matter how big the circle is, one complete rotation of the circle always touches at a little past 3 diameters: at $\pi$.

Animation by John Reid. Copy and paste in your browser- *https://en.wikipedia.org/wiki/File:Pi-unrolled-720.gif#/media/File:Pi-unrolled-720.gif*

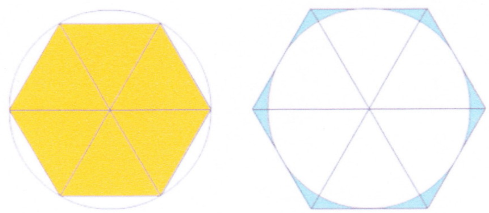



A way to imagine this is to pretend that the red circle around the orange wheel at the start is a red tire. As the wheel rolls it unrolls the tire and leaves it unrolled in a straight line on the road. Now if you measure that unrolled red tire in the road, using the diameter of the wheel as 1, the unrolled tire is 3.14159.... diameters long.

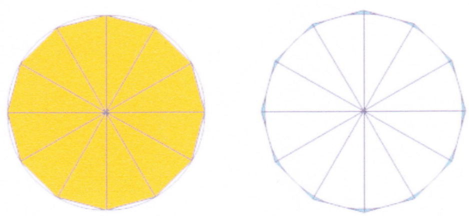
Here is a link for another way to see this, if you are interested. Otherwise, read on.
Pi, Where does it come from? *https://www.youtube.com/watch?v=TlY-Sh9Rzas*

The Babylonians used $\pi = 3$ for practical computation. But one 3,800-year-old Babylonian mathematical tablet, written in cuneiform and excavated in 1936 near Susa, Iran about 260 miles from Babylon, gives a better approximation of $\pi \approx \frac{25}{8}$.

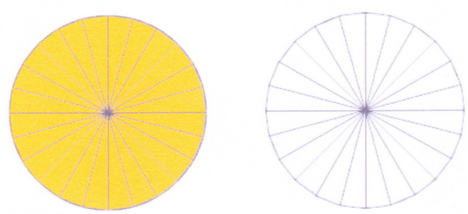
This gives $\pi \approx 3\frac{1}{8}$, or 3.125 in decimal fraction notation, which is really close to $\pi = 3.1416$ that is the practical number for Pi we use today.



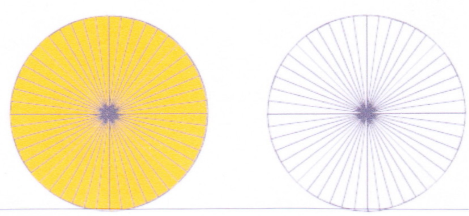
**6-Sided Polygon**
inscribed perimeter = 3.0
circumscribed perimeter = 3.4641



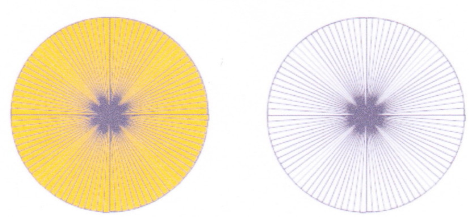
**12-Sided Polygon**
inscribed perimeter = 3.1058
circumscribed perimeter = 3.2154



**24-Sided Polygon**
inscribed perimeter = 3.1326
circumscribed perimeter = 3.1597



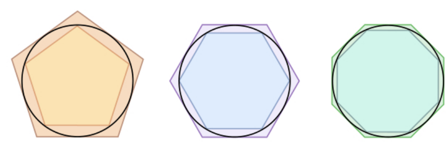
**48-Sided Polygon**
inscribed perimeter = 3.1394
circumscribed perimeter = 3.1461



**96-Sided Polygon**
inscribed perimeter = 3.1410
circumscribed perimeter = 3.1427

Do you remember how the Babylonians drew a hexagon inside a circle? One way they found an approximate value of Pi was to measure the perimeter of the hexagon, (perimeter is the distance around a shape that is the sum of all its sides). Then they could imagine that this perimeter was pretty close to the circumference (the distance around, like a perimeter) of the circle. And then they could divide that circumference by the length of the diagonal (the line that goes from corner to corner of the hexagon), which also is the diameter of the circle, to find an approximate value of π.

They could get closer and closer to the actual value of the circumference by drawing regular polygons (*poly*- from a Greek word meaning many and -*gon* meaning angle; a hexagon is a six-angled, six-sided shape) inside the circle with more and more sides. So instead of a hexagon with six sides they could draw a polygon with 8 equal sides, or



12 sides, 24 or 48.
<——————
About 2,270 years ago Archimedes, a Greek mathematician, doubled the number of sides of the hexagon to a 12-sided polygon, then a 24-sided polygon, and finally 48- and 96-sided polygons. By doing this he was able to bring the perimeter closer and closer in length to the circumference of the circle. After finding the perimeter of polygons with 96 sides inscribed inside a circle and circumscribed outside it, he determined that Pi was greater than 3 and 10/71 but less than 3 and 1/7. In the decimal notation we use today, this translates to between 3.1408 and 3.1429. That's pretty close to the value we now use for Pi as $\pi$ = 3.1416.

Interested students can go to this link to see how Archimedes estimated the value of Pi: *https://www.youtube.com/watch?v=_rJdkhlWZVQ*

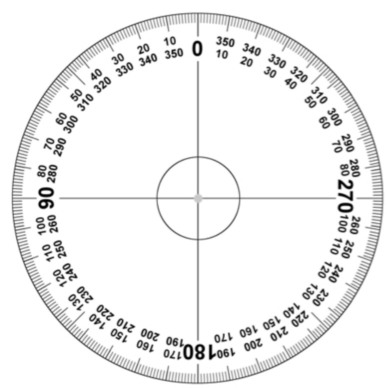
*Liu Hui on a stamp*

About 1,755 years ago, around 265 AD in China, the Wei Kingdom mathematician Liu Hui used a 3,072-sided polygon to find a value of $\pi$ = 3.1416.

A similar approach, with this step-by-step attitude toward solving problems, eventually led to the development of the calculus 1,900 years later. Around the year 1675, Isaac Newton and Gottfried Wilhelm Leibniz independently developed the theory of the calculus. In this frame of mind you repeatedly refine a process over and over with each repetition getting closer and closer to what you are looking for.

Pi is an interesting number because its decimal goes on without end. The first 52 numerals of $\pi$ = 3.141592653589793238462643383279502884197169399375105.... The dots ... after the last 5 means that there are more numerals that have been left out. In fact, using computers we have found trillions of the numerals that follow after the decimal point. But for most purposes we can round up the first 5 (3.141**5**9...) to 6 (because there is a 9 after it), and use $\pi$ = 3.1416 as the practical value for Pi.

In basic mathematics, Pi is used to find the area and circumference of a circle. You might not use it yourself every day, but $\pi$ is used in calculations for quantum physics, air travel, building and construction, communications, medical procedures, music theory, and space flight. It is used for statistics, fractals, thermodynamics, mechanics, astronomy, number theory, trigonometry and electromagnetism. Pi is used in solving problems for electrical applications, and for navigation and GPS location.

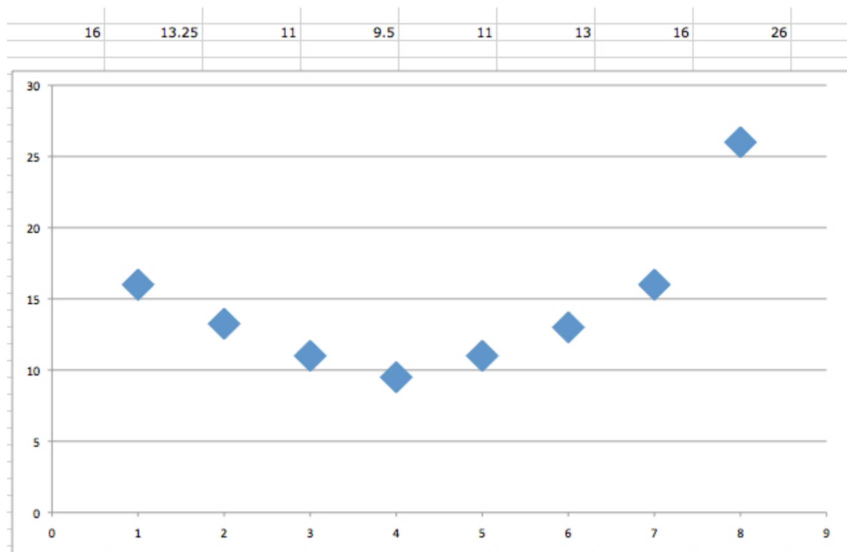



It is used to measure circular velocity of things like truck wheels, motor shafts, engine parts, gears, and it is used to measures voltage across a coil and a capacitor, the wavelength and frequency and amplitude of ocean waves, light waves and sound waves, (One full sine wave is completed in $2\pi$.), river bends, radioactive particle distribution, and the way springs bounce, pendulums swing and strings vibrate.

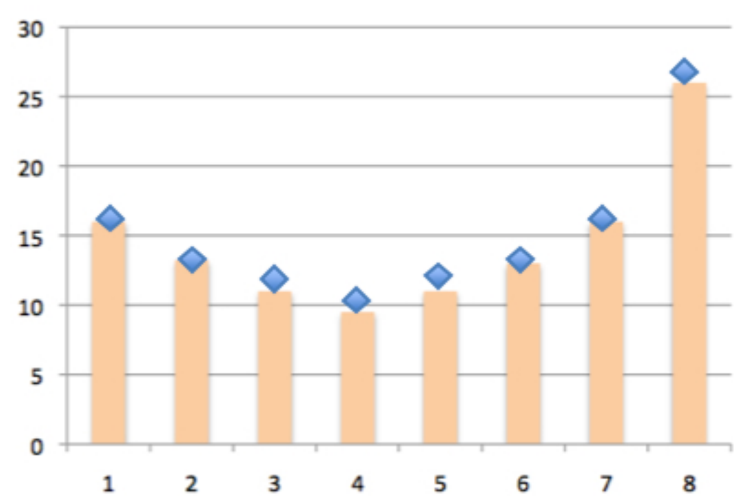
Pi is used to analyze the probability of anything happening, and to track population dynamics. The DNA helix is held together by Pi-bonds. If we measure the total length of any river in the world, including all of its bends and curves, and divide by the length of a straight line from its source to mouth, it averages to $\pi$. Pi can be found in a rainbow, in the moon, in the sun, in the pupil of the eye and a raindrop.

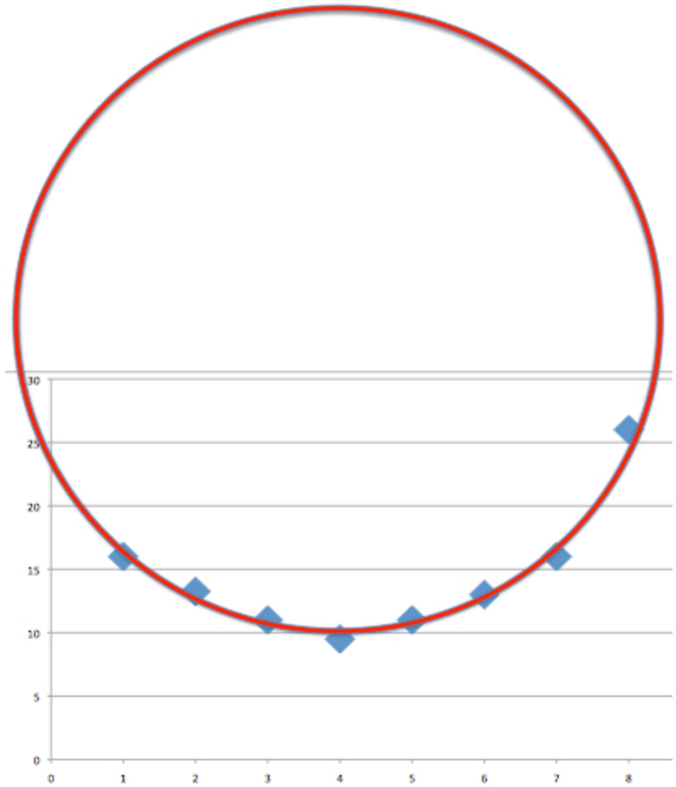
Here's a Song About Pi: *https://www.youtube.com/watch?v=nJkwlnN7VII*

So, what is the connection between circles, $\pi$, 360 degrees, and a sundial? Well, do you remember the lengths of the shadows from your sundial data table? The lengths in inches of the shadows from my data table were 16, 13.25, 11, 9.5, 11, 13, 16, and 26. Below is a graph with marks for each hour across the bottom [This bottom line is often called the positive x-axis.], and marks for inches up the left side [This vertical line is often called the positive y-axis.]. The blue diamonds mark the length of each shadow for each hour that I measured. The first hour that I measured a shadow it was 16 inches long. So the diamond above the 1 is a little bit above the 15-inch line. I measured shadows for eight hours, so there are eight diamonds on the graph.



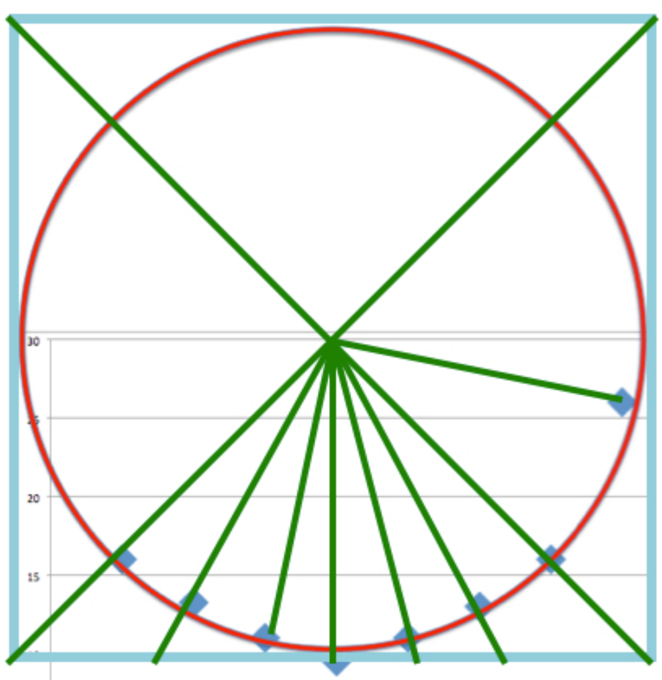

The orange bars on this graph are the same as the bar graph that I made from the shadow lengths from my sundial data sheet, except those bars had different colors. You can see that the blue diamonds are just the points at the tops of the bars. So just as the bar on my first hour shows the length of the shadow, the blue diamond also shows the length of the shadow.






Something interesting is that we can draw a circle that touches each of the diamond data points for the lengths of the shadows. This works on any graph for lengths of the sun's shadows each hour, as long as the vertical axis (the y-axis) is not stretched out so long that the data points start forming a parabola shape.

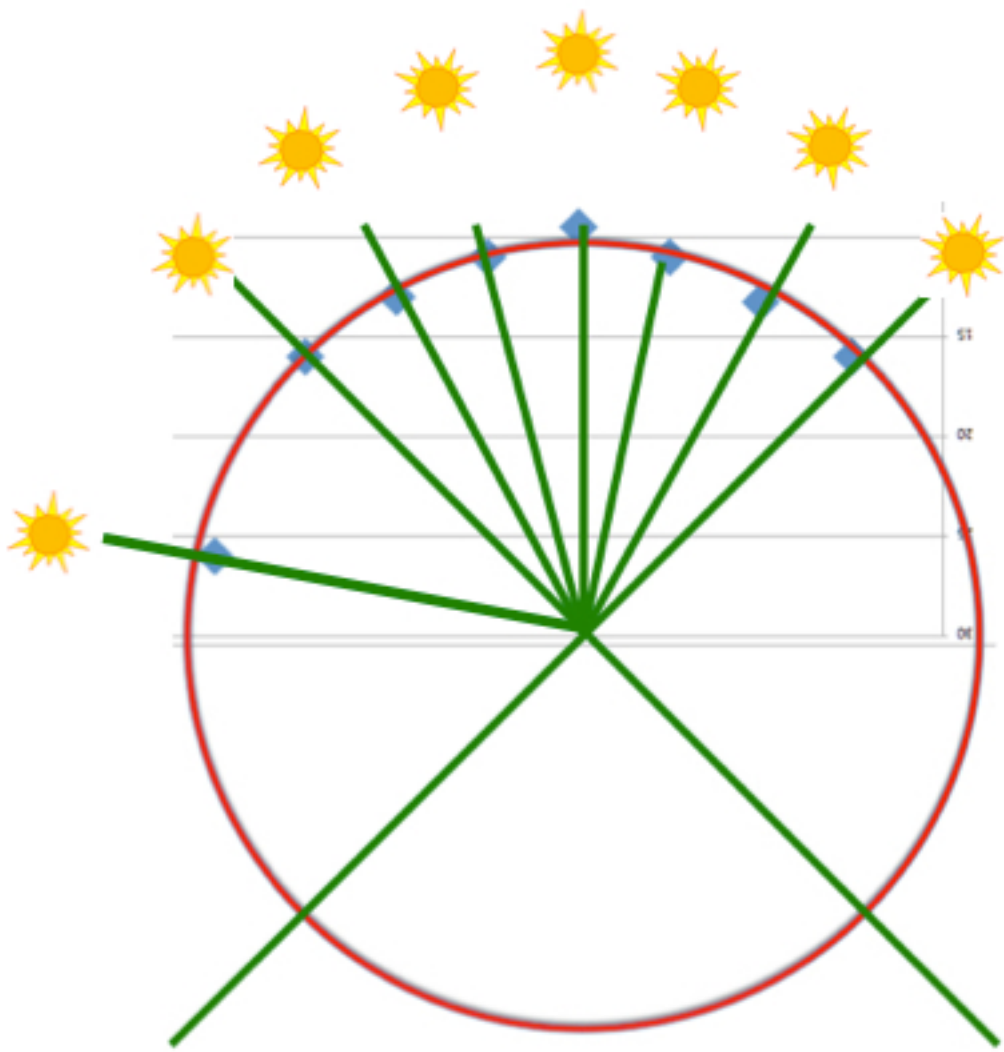
We can find the center of the circle by drawing a square around it so that all four sides of the square touch the circumference of the circle. Then we can draw lines on both diagonals of the square (the opposite corners). Remember, diagonal is from a Greek word *diagonios* meaning from angle to angle (or corner to corner). The point where the diagonals cross is the center of the circle.




Then we can draw lines, or rays, from the center of the circle through each of the data points that represent the lengths of our sundial shadows.

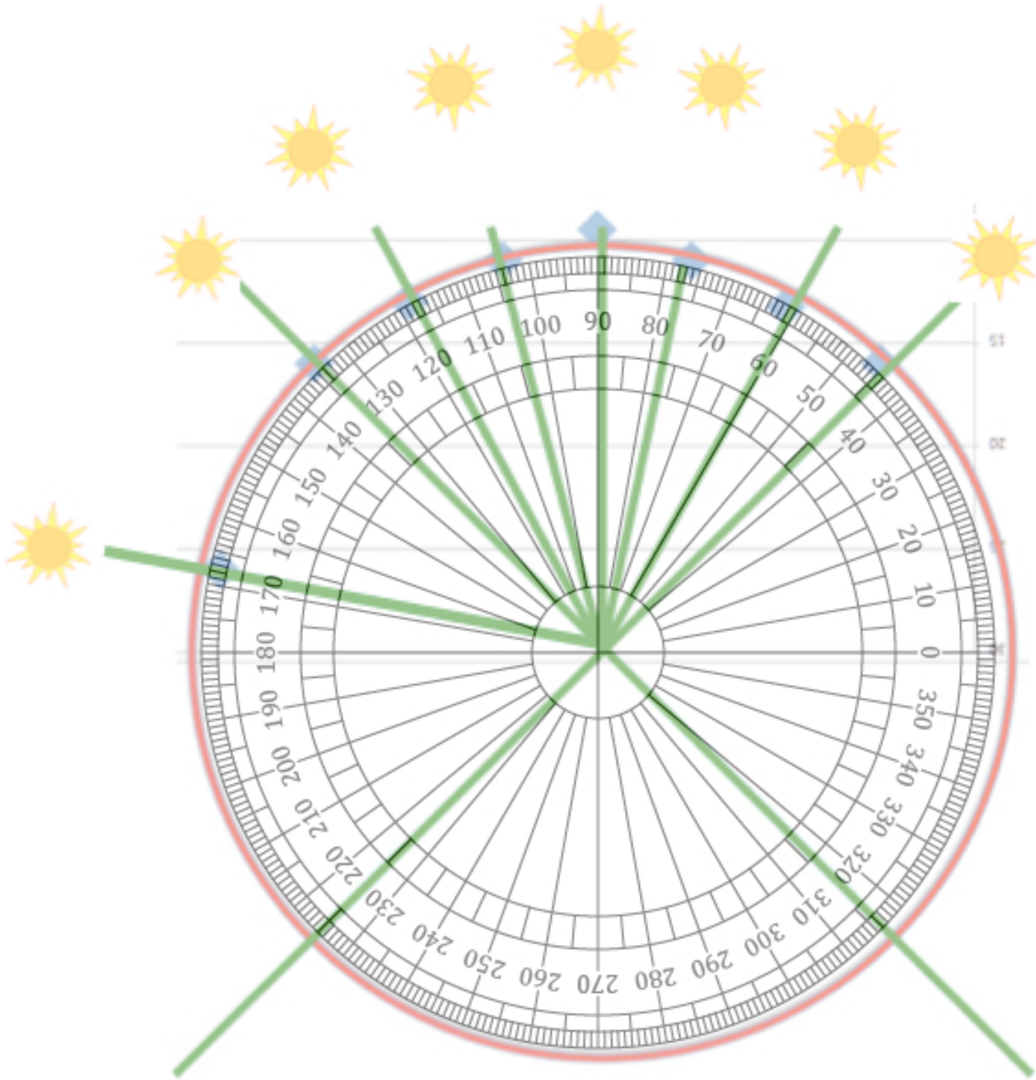
Now imagine and pretend you are a Babylonian astronomer and mathematician, and you turn the graph upside down, and maybe the lines pointing down on the graph for the shadows on the sundial now point up to what looks like the position of the sun every hour, on the day we measured the shadows, as it seemed to travel across the sky from east to west. This might be what the Babylonians imagined 5,000 years ago. What do you think?

Here's what this looks like looking toward the north from 9:00 o'clock in the morning to 4:00 o'clock in the afternoon, with the sun rising in the east, on the right, in the morning, and setting in the west, on the left, in the afternoon. This looks a bit like a clock, doesn't it?



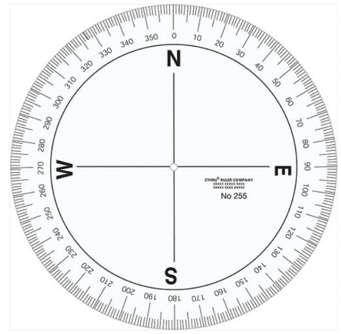
[You can download and print different kinds of measuring tools at this website: *https://www.printablerulers.net/*]

Finally, those creative Babylonians could overlay their 360-degree circle, now used as a measuring tool called a protractor, on the upside-down graph. So they could tell in degrees how far the sun seemed to travel at different times of the day, and also where it was located in the sky at any time.



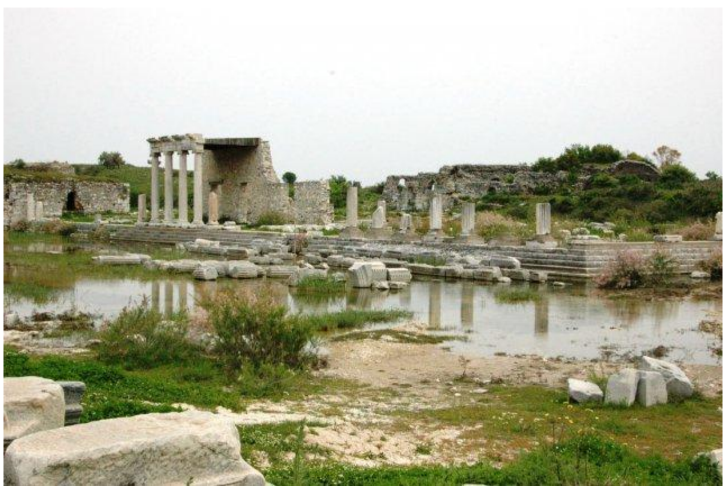

Maybe someone thought that if you held the 360-degree circle parallel to the ground it might be handy for finding directions and locations on earth, as well. But this idea wasn't very useful because a small twist to the left or right would make the directions wrong. People didn't figure out how to do this for a very long time until the invention of a useful compass. We usually think of an invention as happening in a short while, at most a few years from an idea to making it real. But some inventions develop in small steps over hundreds of years, the idea passing from one person to another.

And in the case of a direction-finding compass it took almost 5,000 years from the 360-degree, Babylonian circle until the making of a compass that used a 360-degree circle for navigation.




The word 'compass' comes from a Latin word meaning measure. Many years ago, two of the meanings of the word compass were circle and sundial! And the word 'circle' is from a Latin word meaning perfectly round figure.

The invention of a compass started with the discovery of magnetism that tradition says was first mentioned about 2,500 years ago by the Greek mathematician, astronomer and philosopher Thales of Miletus. We don't know for sure, because nothing he wrote has survived. But he was super famous, and lots of other famous people, like Aristotle, told stories about him. Miletus was a Greek port city on the coast of Anatolia in what is now Turkey. Almost everybody knew about Miletus. It even was mentioned in the Bible.



<--(*the ruins of Miletus*)

Thales lived at a time, 2,500 years ago, when only a few people were just beginning to write a lot of things down, like stories and histories, plays and ideas. Before that time, most of the things that people knew or thought about were passed on to other people by talking while doing things, or telling stories out loud. So people had to remember all these things, like how to cook or make boats, or long stories about what happened many years ago, or what plants could be used for medicine, or how to make a toy or play a game. Do you think you could remember all the stuff you need or use for everything you do, if nobody wrote it down?

One of these stories is about the discovery of magnetized magnetite. Magnetite is a black or brownish-black mineral with a metallic shine that is attracted to a magnet and that can be magnetized to become a magnet itself. It is the most magnetic of all the naturally occurring minerals on Earth. Naturally magnetized pieces of magnetite, now called lodestone, will attract small pieces of iron, which is how ancient people first discovered magnetism. Some scientists think that lodestones may be magnetized by the strong magnetic fields surrounding lightning bolts. *Lodestone* is a 1,000-year-old English word meaning the stone that leads, from the ancient meaning of *lode* as 'journey or way.'

About 500 years after Thales, a Roman writer called Pliny the Elder [who wrote probably the first encyclopedia that was a series of 37 books on natural history called *Naturalis Historia*], repeated an old legend about Magnes the shepherd discovering magnetized magnetite while on the sacred Mount Ida, about 200 miles north of Miletus, but only about 40 miles from the ancient city of Troy. [The Greek poet Homer told an amazing story about Troy called the *Illiad* that you'll probably read when you're older.] This is what Pliny the Elder wrote:

"*magnes appellatus est ab inventore, ut auctor est Nicander ... invenisse autem fertur clavis crepidarum, baculi cuspide haerentibus, cum armenta pasceret*.'

[That's what writing in Latin looks like, and this is what it says in English.]

'Nicander advises that it [magnetized magnetite] was called magnes after its finder ... and indeed is reported to have discovered it clinging to his sandal nails and staff tip, when grazing his herds.'

To the ancients, rocks could be weird." (D. W. Emerson (Systems Exploration (NSW) Pty Ltd) (2014) The Lodestone, from Plato to Kircher)

Most people now think that magnetite was named from the ancient Greek phrase [This is what Greek writing looked like. ------------->] μαγνῆτις λίθος (*magnētis lithos*) meaning stone from Magnesia, referring to the region on the Aegean coast near the city of Magnesia (*ruins of Magnesia*)-> in present-day Turkey where these magnetic lodestones were found. The city of Magnesia was only about 15 miles from Miletus.

Some of the first coins in the world were made 2,600 years ago in these two cities.

This is a coin from Miletus.

This coin has a lion.

And this is a coin from Magnesia.

And this coin has a bull.

Images of lions and bulls were important to the ancient Greeks. Lions were imagined as guardians and protectors, and often lion statues were found guarding the gates to cities. This is one of the stone lions that guarded Miletus.

Both of the cities of Magnesia and Miletus were abandoned and fell into ruins at about the same time around 1,700 years ago, probably because they were destroyed in wars and floods.

At about the same time, 2,500 years ago, that the Greeks were curious about lodestones people in India and China were experimenting with them, too. The Chinese found out that when a lodestone hangs from a thread it always points in the same direction. About 500 years after Thales they started using lodestones to line up their buildings with the north or south. They also used them for fortunetelling. It took another 1,000 years of messing around with lodestones to develop a compass that could be used for finding directions for travel.

Around the year 600, a Chinese experimenter discovered that stroking a thin piece of iron or a needle over and over with a lodestone would magnetize the iron or needle. And about 400 years after that, around the year 1,000, the Chinese figured out that they could float the magnetized iron or needle on water in a bowl. And it always lined up pointing north and south. A Chinese manuscript written in 1040 mentioned an iron fish floating in water that pointed to the south, like this one in a bowl. -----------------> Sometimes the needle could be floated on a piece of cork or light wood.


Now they had something really useful. They had the first compass! But it was hard to use. In the year 1088, Shen Kuo wrote that when "magicians rub the point of a needle with lodestone, then it is able to point to the south…It may be made to float on the surface of water, but it is then rather unsteady…It is best to suspend it by a single cocoon fiber of new silk attached to the center of the needle by a piece of wax. Then, hanging in a windless place, it will always point to the south."

Sailors really wanted a compass, because if they had one they could lose sight of land and still not get lost at sea. The needles hanging from threads were okay for land, but on a ship they swung around all the time. So ships used the needles floating in a bowl of water. Even if the ship tipped from side to side, the water and needle in the bowl stayed level. But it still could bob, or splash out of the bowl. Then in the late 1200s, the magnetic needle was set on a pivot rod standing on the bottom of a compass bowl with a cover, so it could only turn in a circle.


This is a painting of the Chinese explorer Zheng He and one of his gigantic ships that at the time were the biggest ships in the world. Starting in the year 1405, he used the Chinese compasses to lead a fleet of 317 ships with 28,000 sailors on the first of seven major ocean voyages from China throughout Asia, and to India, eastern Africa, and the Middle East. Nothing like this had ever happened before.

In the year 1519 the Portuguese explorer Ferdinand Magellan lead a fleet of five ships, using improved compasses, on a voyage around the world. Three years later, in 1522, the only remaining ship of the fleet, named Victoria, made it all the way back home, becoming the first ship ever to sail all the way around the world.

It was the magnetic compass that helped make this possible.

This is a painting of the Victoria. ---->

**PROJECT**

You can **make a Chinese water compass** using things you have around your house!

You will need:

1) a medium sized sewing needle, about 2 inches long; or thin steel wire, or a 2 inch piece of straightened (small) paper clip [if you have pliers and wire cutters]
2) a magnet; even a refrigerator magnet will work.
3) a floater: something to float the needle on, like a thin piece of cork, wax paper, even plain paper, a thin piece of light wood (like balsa), a plastic bottle cap, a leaf, or other ideas you may have for floaters
4) a bowl, not too deep and at least 5 inches or more wide; an aluminum pie pan or glass pie plate is good, half-filled with water
5) a toothpick or two
6) scissors

**Step 1**. Hold the needle by its eye, or put it on the table or the floor pressing your finger on the eye, and stroke the magnet along the needle's length 50 to 100 times, always in one direction (not back and forth) and always with the same end of the magnet. This magnetizes the needle so it becomes a pointer. Do the same if you're using a wire or piece of paper clip. [Now, if you want, you can color one tip with a permanent marker, but you don't have to.]

**Step 2**. Your floater should be light and have a diameter or length that sort of fits your pointer needle or wire (approximately, not exactly). So if you're using wax paper or regular paper, or craft foam, or something else that you can cut with scissors, now is the time to cut a circle that has a diameter of about 2 inches (You remember a diameter from page 22.). If you are using wood or cork or a bottle cap, it could be smaller than 2 inches in diameter.

**Step 3**. Take your bowl or pie plate to a place that is **not** close to anything that is metal, like the refrigerator, and fill it about half full of water. Then wait until the water stops moving.

**Step 4**. Carefully place your floater in the center of the bowl of water, trying not to get water on top of it. You can gently drop it when it is really close to the water. You can take it out and try again, if you don't like how it's floating.

**Step 5**. This is the tricky part. Now very gently put your magnetized pointer needle or wire on top of your floater, without pushing the floater into the water. If that's not working you can try over by gently dropping your needle or wire when it is really close to the top of your floater. Once your pointer is on the floater, you can use the toothpick to gently nudge it closer to the middle.

If even that doesn't work, you can take the floater out of the water, dry it off, and tape the needle or wire to it, or glue it with a teeny-tiny drop of glue. Then, gently put your floater with the taped-on or glued-on pointer into the water. [The only problem is that sometimes tape won't hold or you have to unglue the pointer to use it with a different floater, or use another needle.]

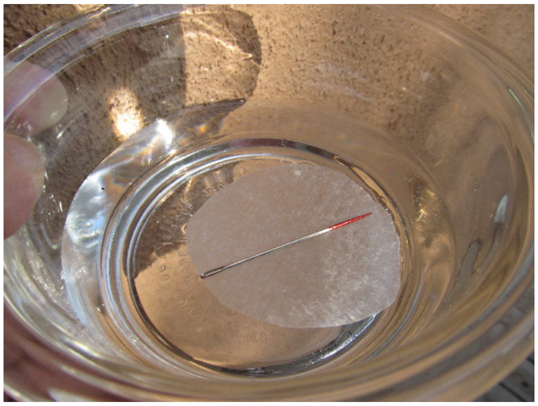
**Step 6**. Watch patiently as everything settles down and your pointer turns slowly (rotates, like the Earth; remember your belly gnomon from pages 8-9) until it stops rotating (turning). Now your pointer is lined up with the magnetic poles of the Earth. One end of your pointer is aiming toward the north, and the other end is pointing toward the south. Every time your pointer settles down it always will point in the same north-south directions.

This is why the magnetic compass was such a handy tool for sailors. With it they could tell which direction they were going, even if they couldn't see land or stars.

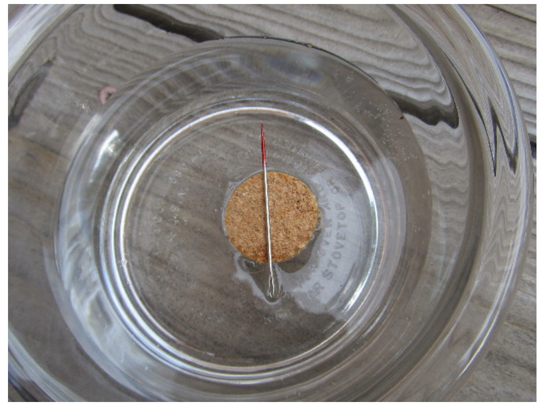
**Tip**. You can use the toothpick to keep the floater and pointer away from sticking to the edges of the bowl. Sometimes if the floater is moving toward the edge of the bowl, all you have to do is put the toothpick in its way, so it bumps into it. Everything you do with your water compass should be gentle.

[A magnetic compass works because the Earth's magnetic field holds the pointer in place in one direction. The north and south poles of the Earth's magnetic field are pretty close, but not exactly, to the geographical North Pole and South Pole of the Earth. The geographical poles are the north and south ends of the axis on which the Earth rotates. But we'll leave investigating all this pole stuff for another project!]

Here are some photos of my water compasses.  All of them worked just fine.



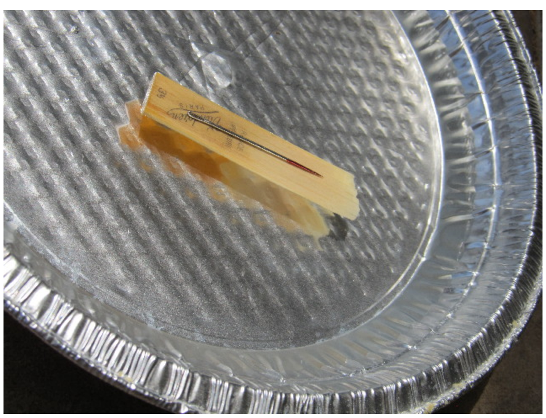
My needle pointer on wax paper



My needle pointer on a piece of cork

Paper clip wire pointer on a bottle cap

Paper clip wire pointer on wax paper



Needle pointer floating on a worn out, old saxophone reed in a pie pan

Needle pointer floating on a leaf in a camping food container

Whenever I go camping in the mountains, I always take a sewing needle and thread.  I make sure the needle is magnetized before I go, just in case I lose or break my compass.  Then I can float the needle in an empty food container on a leaf from a tree or bush, and I know which direction is north, south, east or west.  And because I also take a sheet of paper with a big 360-degree circle printed on it (like the one on page 30), I can see all the angles in between, so I can line them up with my map.

**Experiment 1**. After your water compass's magnetic pointer has stopped turning, and is pointing north-south, slowly rotate the bowl while watching the pointer. With the bowl sitting wherever you have put it, rotate the bowl slowly by holding its edges and turning it gently around a little bit at a time, without picking it up. What is the pointer doing?

**Experiment 2**: Gently lift up one side of your water compass bowl while watching the pointer. Be careful to not lift it so high that the water spills. What is the pointer doing? Now gently lift it from the other side. What is the pointer doing? What is the water level doing? Can you see why sailors thought a water compass was handy to have on a ship that rocked side to side?

**Experiment 3**: Pick up your water compass and take it to another place in the house. Is it pointing in the same direction? Now take your water compass outside. After it settles down, is it pointing in the same direction as it was in the house?

**Experiment 4**: Try using different kinds of floaters in your water compass. Do you think one type works better than the others? Why? Can you rank order the floaters? That means can you list the floaters you tested in order from the best to the worst?

| | |
|---|---|
| #1 the best floater is | Why it's the best: |
| #2 the second best is | Why: |
| #3 the third best is | Why: |
| #4 the worst floater is | Why: |

**Experiment 5**: Try using different kinds of magnetized pointers on your best floater. Does one type of pointer work better than others? Which one was it?

**Experiment 6**: Using your best floater and starting with an unmagnetized copy of your best pointer, see what happens the more times you stroke the pointer with the magnet. Remember, always stroke the pointer with the magnet in one direction (not back and forth) and always with the same end of the magnet.

Let's go Babylonian base 60 and start with 15 strokes, then test the pointer on your floater. Next, take the pointer out of the bowl and add 15 more strokes for 30 total. Then test it on your floater. Next add 30 more strokes for 60 total. Test. Add 60 more strokes for 120 strokes total. Test. [Notice that we're doubling the number of strokes every time.] Were there any differences in how the pointer worked each time you added more strokes of the magnet?

| | |
|---|---|
| 15 strokes of the magnet | Test Result: |
| 30 strokes | Test Result: |
| 60 strokes | Test Result: |
| 120 strokes | Test Result: |

This Chinese compass with a sundial was an improvement after the water compass. Zheng He might have had something like one of these, on his voyage to Africa.
**You see; we're back to sundials where this project started!** Everything comes together when you try to sail across an ocean: time, direction, circle geometry, angles, degrees, numbers, sun and stars. You need to use them all.

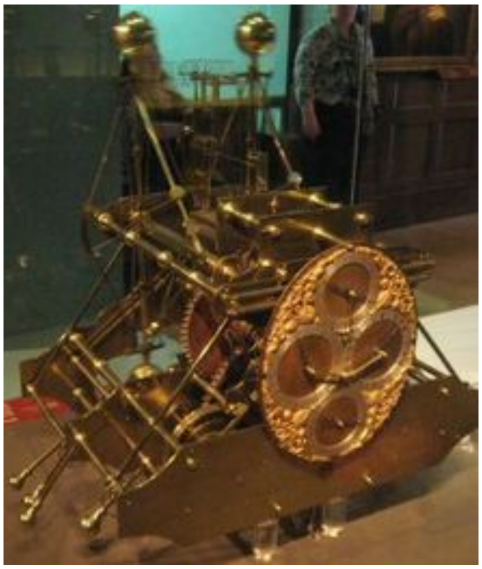
This Chinese navigating tool has a compass in its wooden base and a hinged sundial. The sundial has a folding gnomon on top and a hinged support underneath that can be put in one of 13 notches in the base to change the angle of the sundial.

Why was it so important for sailors to tell time along with a compass to tell direction? Well, the compass could tell them what direction their ship was pointing, but it couldn't tell them if a wind or current was pushing them sideways, or how far they had gone.

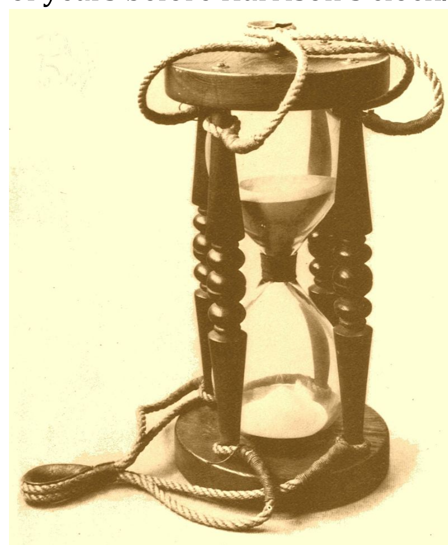
But they could align their sundial's gnomon shadow with an hour mark at sunrise on the day they left home and measure the angle of the sun from the horizon a few hours later. Then they could do the same thing from their ship one or more days later. By finding the difference in the angles of the sun from one day to another and comparing it to records of these differences kept by many ships, and also knowing about how fast their ship has been going by using a log line, and knowing how many hours it had been since they left home by using a sand glass (an hour glass), they could estimate approximately how far east or west they were from their home.

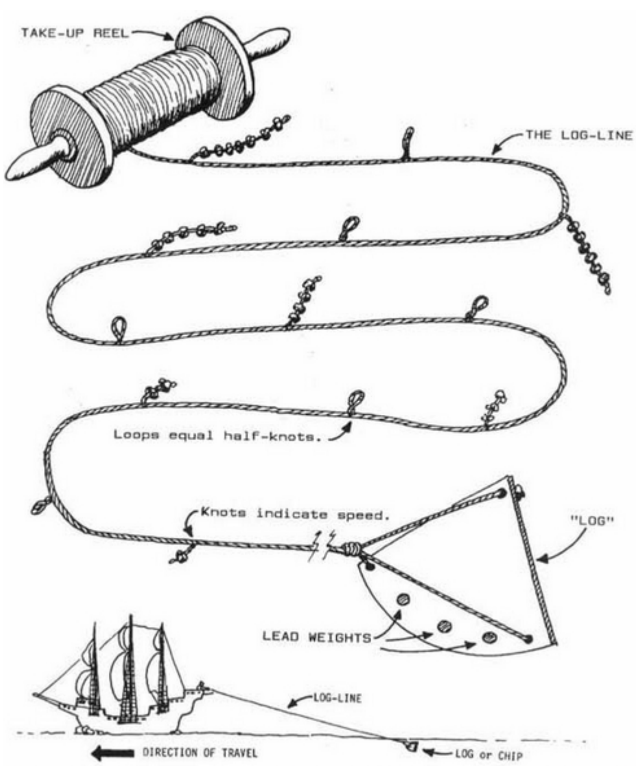
The ancient sailors also could use angle finders like a cross-staff or astrolabe to measure the angle of a star or the sun from the horizon and by comparing this angle to the angle they measured at home, they could tell approximately how far north or south they were from where they started. These ways of figuring out where they were just gave them a good guess, but were not very accurate until the development of a very accurate ship's clock by John Harrison in 1759. It took him 31 years to finish inventing a clock that could keep accurate time on a long voyage of a ship, a clock that was not affected by changes in temperature, air pressure or humidity, remained accurate over long time intervals, resisted corrosion in salt air, and was able to function on board a constantly-moving ship

This was John Harrison's first try at a ship's clock in 1736. It didn't work.



This is a sand glass, or hour glass, used to tell time on ships for hundreds of years before Harrison's clock.

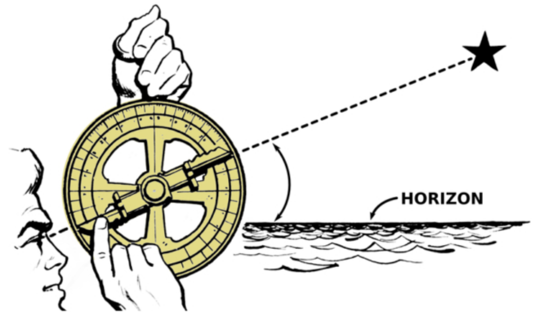




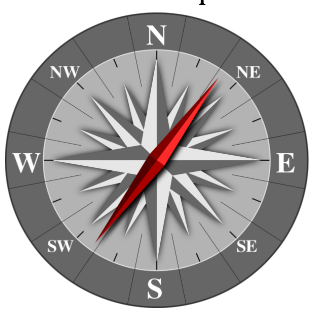

This is a log line, which could be used with a sand glass to tell how fast a ship was going. Sailors would throw the 'log' (originally a real log) into the water in back of the ship. The log was tied to a long rope with knots tied in it every 7 fathoms (42 feet). One fathom = 6 feet, and 'fathom' comes from an Old English word meaning outstretched arms. The friction of the water would keep the log in one place, and as the ship sailed away the sailors counted how many knots on the rope went over the side of the ship in half-a-minute. They had a half-minute sand glass. Then they could calculate how many knots a minute, or knots per hour, they were traveling. Ships and planes still tell their speed in knots.

This is a cross staff used by sailors 600 years ago to tell the angle of the sun or a star from the horizon.

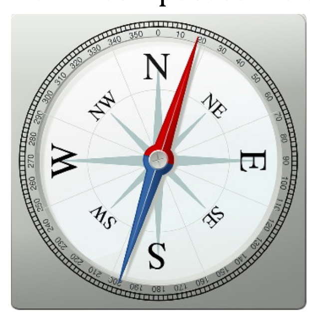
This is an astrolabe that was more accurate than a cross staff to find the altitude of the sun or star.



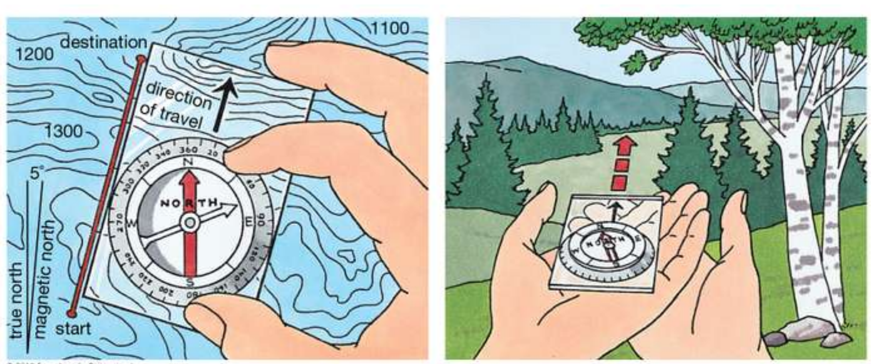

This is called a compass rose, also called a windrose, that was used for hundreds of years on ships to show the directions of the main winds on the ocean and the direction the ship was sailing.




It was only 150 years ago that the 360-degree circle was added to ships' compasses. Finally, everything from the Babylonians to the 1900s came together in compasses we use today.




These are drawings of a camping compass that is made to use with a topographic map to find your way in the wilderness, mountains or national parks. Topographic maps show mountains and valleys and how tall or deep they are. The government's Geological Survey makes these maps of everyplace in the United States, and you can see them at the following website: *https://viewer.nationalmap.gov/basic/?basemap=b1&category=histtopo,ustopo&title=Map%20View*



This is what a U.S. Geological Survey topographic map looks like.


Here is a way-finding compass being used on a USGS topographic map.


Do you think you could find your way or your location with a map and a compass?